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GENERATION OF AN ELECTROMAGNETIC FIELD IN THE NONSYMMETRIC RAILS AND PROTECTIVE ARMATURE OF A RAILGUN UNDER AN INCREASING CURRENT PULSE

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The calculation of the electromagnetic field in a system of conductors under a powerful short-time current pulse for a time of the order of 1 msec is very important for various applications, in particular, for the design of powerful electromagnetic launchers (railguns). Here it is important to know the distributions of the electromagnetic field and heating in the conductors, which depend on the geometry and other parameters of the system. It is also important to know the forces on the conductors due to the electromagnetic field. In particular, the inductance per unit length in the system [1] is of significance.

Various methods have been developed for the problems of calculating distributions of magnetic-field, heating, current density, and other parameters for a system of infinitely long conductors. These problems, however, have been solved or have relatively simple solutions for steady (direct) current or alternating high-frequency current. In the former case, it is necessary to solve a steady Poisson equation. In the second case, the approximation of a thin skin-layer in which current flows over the surface of conductors is operative [2]. If we consider a single current pulse whose duration is comparable with the characteristic time of magnetic-field diffusion into the conductors, the electromagnetic-field distribution differs considerably in both the former and the latter cases and varies in time greatly. In particular, launcher characteristics such as inductance and resistance per unit length vary in time as well.

In the present paper, we construct a numerical model for calculating the electromagnetic field in a system of infinitely long conductors of an arbitrary profile which carries a time-varying current. A similar formulation of the problem was realized in [3], but in this paper, we give the most general formulation. We intend to find the electromagnetic-field distribution for an arbitrary (not necessarily symmetric) profile of a system of conductors separated by a dielectric. The system carries a current that arbitrarily varies with time and is different in magnitude and direction for different conductors. If a strong current flows in opposite directions in two conductors which are surrounded by a third conductor with a zero total current, which is a protective armature, the system models an electromagnetic railgun. This formulation of the problem is inadequate to model a real launcher, because the current is essentially three-dimensional. The problem was solved in three-dimensional geometry, for example, in [4–6]. A two-dimensional calculation, however, makes it possible to estimate the efficiency of one or another rail profile and the current distribution at a low cost of computer time, and is, therefore, very important for optimization problems. In addition, the numerical method proposed herein for solving the two-dimensional problem in the general formulation can be used not only for the calculation of rail launchers.

Physical Formulation of the Problem. A system of several conductors infinite along a straight line is considered. As applied to the calculation of electromagnetic launchers, these conductors can be treated as rails, a protective metallic armature, and an inductance coil [1, 7, 8]. The conductors are isolated by a dielectric, and the entire system is also placed in the dielectric. In the cross section of the system, one conductor must not border on another conductor. Each of the conductors carries a time-dependent current

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which is specified as an input parameter of the problem. The initial magnitude of this current is equal to zero. Zero magnetic induction and a uniform temperature distribution are initially assumed. At each time it is required to find magnetic-field, electric-field, current-density, and temperature distributions (assuming that the thermal energy is released owing to the Joule heating of the conductors), the distribution of the volume electromagnetic forces that act, due to the magnetic field, on the current-carrying conductors, and also integral parameters, such as resistance and inductance per unit length. Thus, we wish to realize the two-dimensional unsteady model in the general formulation.

Mathematical Formulation of the Problem. Let us introduce the system of coordinates $Oxyz$, assuming that all conductors of the system are aligned with the Ox axis. The Oy and Oz axes are located in the plane of the cross section. Without loss of generality, the coordinate origin can be located at any point that is fixed with respect to the system.

For the Maxwell equations in the magnetohydrodynamic (MHD) approximation, we introduce a vector potential A and a scalar potential Φ by the following formulas [9]:

$$B = \text{rot } A, \quad \text{div}\, A = 0, \quad E = \text{grad } \Phi - \frac{\partial A}{\partial t}. \tag{1}$$

Here B is the magnetic-induction vector and E is the electric-field-intensity vector. Taking into account the Ohm law (neglecting Hall current), from the Maxwell equations and (1) we obtain

$$\nabla^2 A = \sigma\mu_a\Big(\frac{\partial A}{\partial t} - \text{grad } \Phi\Big); \tag{2a}$$

$$\nabla^2 \Phi = 0; \tag{2b}$$

$$j = -\nabla^2 A/\mu_a \tag{3}$$

(j is the current-density vector, μ_a is the absolute permeability of free space, and σ is the conductivity of the material).

The conditions $B \to 0$ and $E \to 0$ are considered boundary conditions for y and z tending to infinity. Therefore, without loss of generality, we set $A \to 0$ and $\Phi \to 0$ for the electromagnetic potentials. Below, we show that the scalar potential can be considered continuous, and the vector potential can be considered continuously differentiable on the boundaries of the conductors.

Assuming that the current flows only along the Ox axis over the entire length of the conductors, for the current density in the conductors we have

$$j = j_x e_x. \tag{4}$$

Here e_x is a unit vector along the Ox axis.

It follows from relation (3) with allowance for (4) that the components A_y and A_z of the vector potential in both the conductors and dielectrics obey the Laplace equation. Taking into account continuity of the potentials at the boundaries of the conductors and the boundary conditions at infinity, we find the solution $A_y \equiv A_z \equiv 0$. Next it follows from the Ohm law and (4) that E in the conductor has a single component E_x, and (1) implies that the condition

$$\Phi = \Phi_k(t, x) \tag{5}$$

must be satisfied in the conductor (k is the conductor number). Since j_x does not depend on x, it follows from (1) and (3) that $\partial\Phi/\partial x$ and A_x do not depend on x either. Therefore, the relation

$$\Phi = \Phi_{0k}(t) + x\varphi_k(t) \tag{6a}$$

must be satisfied in the conductor. For the dielectric, however, using the condition that the scalar potential is continuous at the conductor/dielectric boundary, from (2b) and (6a) we obtain

$$\Phi = \Phi_0(t, y, z) + x\varphi(t, y, z). \tag{6b}$$

It is impossible to find $\Phi_{0k}(t)$ from the available equations, because all equations contain only the gradient of the scalar potential and, hence, Φ_0 cannot be determined uniquely for the dielectric. For this, it is necessary to know the boundary conditions at the ends of the conductors or at two different cross sections of the system. Therefore, the problem of infinite parallel conductors is not closed. However, if we exclude Φ_0 and Φ_{0k} from consideration, we come to a closed problem in which the gradient of the scalar potential $\varphi(t, y, z)$ along the Ox axis is determined uniquely.

Thus, we come to the following working equations and relations for the vector and scalar potentials:

$$\frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} = \sigma\mu_a\Big(\frac{\partial A_x}{\partial t} - \varphi_k(t)\Big); \tag{7a}$$

$$\frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0; \tag{7b}$$

$$A_y = A_z = 0; \tag{7c}$$

$$\varphi = \varphi_k(t) \quad \text{in the } k\text{th conductor.} \tag{7d}$$

For the magnetic induction, electric-field intensity, and current density, we have the relations

$$B_x = 0, \quad B_y = \frac{\partial A_x}{\partial z}, \quad B_z = -\frac{\partial A_x}{\partial y}; \tag{8a}$$

$$E_x = \varphi_k - \frac{\partial A_x}{\partial t}, \quad E_y = E_z = 0 \quad \text{in the conductor;} \tag{8b}$$

$$E_x = \varphi_k - \frac{\partial A_x}{\partial t}, \quad \frac{\partial E_y}{\partial x} = \frac{\partial \varphi}{\partial y}, \quad \frac{\partial E_z}{\partial x} = \frac{\partial \varphi}{\partial z} \quad \text{in the dielectric ;} \tag{8c}$$

$$j_x = \sigma\Big(\varphi_k - \frac{\partial A_x}{\partial t}\Big), \quad j_y = j_z = 0 \quad \text{in the conductor;} \tag{8d}$$

$$j_x = j_y = j_z = 0 \quad \text{in the dielectric.}$$

To close system (7), the conditions at infinity,

$$A_x = 0, \quad \varphi = 0 \quad \text{for } y = \infty \text{ and } z = \infty, \tag{9}$$

should be supplemented by the condition for the passage of the given current $I_k(t)$ through the kth conductor This condition is obtained from relation (8b) and relates $I_k(t)$ and $\varphi_k(t)$ as follows:

$$\varphi_k \int\limits_{S_k} \sigma dS = I_k + \int\limits_{S_k} \sigma \frac{\partial A_x}{\partial t} dS. \tag{10}$$

Here S_k is the cross-sectional area of the kth conductor.

In addition, we assume that the scalar potential and its gradient φ are continuous along the Ox axis, and the potential A_x is continuously differentiable at the conductor–dielectric boundary. Let us show that, with no surface currents, these conditions do not contradict the corresponding conditions imposed on the vectors E and B in the MHD approximation.

Sedov [10] showed that at a fixed boundary between two media, the following relations resulting from the integral Maxwell equations in the MHD approximation must be satisfied:

$$E_{n1} - E_{n2} = \gamma/\varepsilon_a, \quad E_{\tau 1} - E_{\tau 2} = 0, \quad B_{n1} - B_{n2} = 0, \quad B_{\tau 1} - B_{\tau 2} = \mu_a(i \times n). \tag{11}$$

Here the subscripts n, τ, 1, and 2 denote, respectively, the vector component normal to the boundary, the vector component tangent to the boundary, and points on the opposite sides of the boundary; ε_a is the absolute dielectric permeability of free space; γ is the surface-charge density; i is the surface-current-density vector; and n is a unit normal vector to the boundary. It follows from relations (8b) that, provided that the potential Φ and, hence φ, are continuous, the tangent component of the vector E at the conductor–dielectric boundary is equal to $E_x e_x$, and its tangent component is continuous. The normal component can have a discontinuity, and its gradient with respect to x is equal to $\partial\varphi/\partial n$. This quantity determines with accuracy to a constant the surface charge induced in the conductors. If we assume that the vector potential is continuously differentiable at the boundary, the vector B is continuous at the boundary. Hence, conditions (11) are satisfied if the surface currents are absent.

Calculation of the electromagnetic field in the profile plane of a railgun makes it possible to calculate physical effects produced by passage of a high current through the rails.

The temperature of the conductors is calculated from the equation

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \nabla^2 T + \frac{j_x^2}{\sigma}, \tag{12a}$$

where ρ is the density, c_p is the specific heat, and λ is the thermal conductivity of the conductors. For the dielectric, the temperature is calculated as follows (neglecting Joule heat release):

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \nabla^2 T. \tag{12b}$$

As boundary and initial conditions for the temperature, we assume that, as $y \to \infty$ and $z \to \infty$, the temperature tends to the initial temperature, and the temperature is continuous on the boundaries between different computation domains.

The volume forces acting on the conductor due to the magnetic field are given by the formula $f = j \times B$ (here electric forces are ignored in comparison with magnetic forces in the MHD approximation). In our investigation, the volume forces are calculated by the formulas

$$f_y = j_x B_z, \quad f_z = -j_x B_y. \tag{13}$$

By definition (see [2]), the inductance L and resistance R per unit length are calculated by the formulas

$$L = 2E_m/J^2, \quad R = W/J^2. \tag{14}$$

Here E_m is the electromagnetic energy per unit length of the system of conductors, W is the Joule heat output per unit length, and J^2 is the square of the current strength in the circuit. In this investigation, these three quantities are defined by

$$E_m = \int\limits_{\Sigma} \frac{B^2}{2\mu_a} d\Sigma, \quad W = \int\limits_{\Sigma} \frac{j_x^2}{\sigma} d\Sigma, \quad J^2 = \max_k I_k^2. \tag{15}$$

The problem formulated in the general form is complicated, because, for the conductor region, the system of governing partial differential equations is parabolic, and for the dielectric domain, this system is elliptic for the vector potential and parabolic for the temperature. In addition, the integral equation (10) is used in the boundary conditions.

Numerical Solution of the Problem. To solve the problem numerically, we use a discrete grid with a constant spacing and the maximum numbers of grid points N_y and N_z along the Oy and Oz axes, respectively. Each grid cell has an integer index that is equal to the conductor number to which it belongs, or to a number that is larger than the maximum conductor number (to the dielectric number). It is assumed that the conductor–dielectric interfaces run between neighboring grid cells and cannot run inside the grid cells. This gives an error in the location of the conductor boundaries, which can be reduced by grid refinement. To generate the computation domain, we developed a specific program adapted for a personal computer of the IBM PC type. This program locates the conductor and dielectric regions in the computation domain.

An index k_{ij} which determines the conductor or dielectric number is assigned to each (i, j) cell, where $0 \leqslant i \leqslant N_y$ and $0 \leqslant j \leqslant N_z$. The grid step sizes along the Oy and Oz axes are constant. We denote them by h_y and h_z, respectively, and the time step size is denoted by h_t. The conductivity, density, heat capacity, and heat conductivity are assumed to depend only on k.

The vector potential of the electromagnetic field A_{ij} (over the entire computation domain) and the scalar-potential gradient φ_k (only for the conductors) are calculated by the difference scheme

$$H_i\Big(A_{i+1,j} - 2A_{i,j} + A_{i-1,j} + \hat{A}_{i+1,j} - 2\hat{A}_{i,j} + \hat{A}_{i-1,j}\Big) + H_j\Big(A_{i,j+j} - 2A_{i,j}$$
$$+ A_{i,j-1} + \hat{A}_{i,j+1} - 2\hat{A}_{i,j} + \hat{A}_{i,j-1}\Big) + \sigma_k\mu_a\Big(h_t\varphi_k - (A_{i,j} - \hat{A}_{i,j})\Big) = 0, \tag{16}$$

$$\varphi_m \sum_{k_{ij}=m} \sigma_k = I_m \frac{h_t}{h_i h_j} + \sum_{k_{ij}=m} \sigma_k (A_{ij} - \hat{A}_{ij}). \tag{17}$$

Here $1 \leqslant i < N_i - 1$, $1 \leqslant j < N_j - 1$, $H_i = 0.5h_t/h_i^2$, $H_j = 0.5h_t/h_j^2$, and $\hat{A}$ is the vector potential at the previous time level. The boundary conditions for the difference problem are chosen to be zero first-order finite differences of A, which corresponds to a zero magnetic-induction tangent vector to the boundary:

$$A_{0,j} = A_{i,j}, \quad A_{N_i-1,j} = A_{N_i-2,j}, \quad A_{i,0} = A_{i,1}, \quad A_{i,N_j-1} = A_{i,N_j-2}. \tag{18}$$

Note that the values of the vector potential at four corner cells are not used in calculations by formulas (16)–(18). Therefore, these values can be disregarded.

The temperature $T_{i,j}$ was found by the difference scheme

$$H_i\Big(T_{i+1,j} - 2T_{i,j} + T_{i-1,j} + \hat{T}_{i+1,j} - 2\hat{T}_{i,j} + \hat{T}_{i-1,j}\Big) + H_j\Big(T_{i,j+j} - 2T_{i,j} + T_{i,j-1}$$
$$+ \hat{T}_{i,j+1} - 2\hat{T}_{i,j} + \hat{T}_{i,j-1}\Big) + \frac{\sigma_k h_t}{\lambda_k}\Big(\varphi_k - \frac{A_{i,j} - \hat{A}_{i,j}}{h_t}\Big)^2 - \frac{\rho_k c_{pk}}{\lambda_k}\Big(T_{i,j} - \hat{T}_{i,j}\Big) = 0 \tag{19}$$

($\hat{T}$ is the temperature at the preceding time level, λ_k, ρ_k, and c_{pk} are the heat conductivity, density, and heat capacity of the kth conductor or dielectric). The boundary conditions are

$$T_{0,j} = T_{i,j}, \quad T_{N_i-1,j} = T_{N_i-2,j}, \quad T_{i,0} = T_{i,1}, \quad T_{i,N_j-1} = T_{i,N_j-2}. \tag{20}$$

System (16) and (17) with boundary conditions (18) and system (19) with boundary conditions (20) are solved by a symmetric successive over-relaxation method (alternately by rows and columns, see [11]). In each iteration, the calculations are performed in the following order: first (16) is calculated by rows for given φ_k, then φ_k is calculated from (17), then (16) by columns, then again φ_k from (17), then (19) is calculated by rows, and then (19) by columns. Each solution of the system of difference equations (16) or (19) yields systems of tridiagonal difference equations with boundary conditions obtained from (18) and (20), respectively. To solve these equations, we use a reduction method (which is described in a simplified form, for example, in [12]) rather than the standard tridiagonal algorithm. Unlike the standard method, the reduction method is absolutely stable for arbitrary coefficients of a tridiagonal equation, provided its solution exists and is unique.

The condition

$$\frac{|U_A^m - U_A^{m-1}|}{\max|A^m|} + \frac{|U_T^m - U_T^{m-1}|}{\max|T^m - T_0|} < 10^{-4} \tag{21}$$

is used as a criterion for the termination of the iterative process. Here U_A^m and U_T^m are the residuals at the mth iteration step:

$$U_A = \Big(\sum_{i,j} U_{ij}^2 h_i h_j\Big)^2, \qquad U_T = \Big(\sum_{i,j} V_{ij}^2 h_i h_j\Big)^2. \tag{22}$$

Here U is the left-hand side of Eq. (16) and V is the left-hand side of Eq. (19), A^m and T^m are the vector potential and the temperature at the mth iteration step, respectively, and T_0 is the initial temperature.

After iteration at each time step, the current density, magnetic induction vector components, and the volume electromagnetic force vector can be calculated as follows:

$$\begin{gathered}(j_x)_{ij} = \sigma_k\big(\varphi_k - (A_{ij} - \hat{A}_{ij})/h_t\big), \quad (B_y)_{ij} = 0,25(A_{i,j+1} - A_{i,j-1} + \hat{A}_{i,j+1} - \hat{A}_{i,j-1})/h_j, \\ (B_z)_{ij} = -0,25(A_{i+1,j} - A_{i-1,j} + \hat{A}_{i+1,j} - \hat{A}_{i-1,j})/h_i, \quad (f_y)_{ij} = -(B_z)_{ij}(j_x)_{ij}, \quad (f_z)_{ij} = (B_y)_{ij}(j_x)_{ij}.\end{gathered} \tag{23}$$

The inductance and resistance of the system per unit length have the form

$$L = \frac{1}{\mu_a}\sum_{i,j}((B_y)_{ij}^2 + (B_z)_{ij}^2)/\max_k|I_k|^2, \quad R = \sum_{i,j:\sigma_k\neq 0}\frac{j_{ij}^2}{\sigma_{k_{ij}}}/\max_k|I_k|^2. \tag{24}$$

Calculation Results. Symmetric and asymmetric model railgun profiles are chosen for calculation. Each of them consists of two rails (conductors) separated by a dielectric. The conductors carry an equal and opposite strong pulsed current. The conductors are surrounded by a protective metallic armature which is also separated from them by a dielectric. The total current in the armature is equal to zero. The railgun channel is located at the center of the profile between the rails. For both cases, the following initial data are used:

— the size of the profile including the surrounding cells, i.e., the size of the computation domain along the Oy and Oz axes, is 10 cm,

— the initial temperature is 280 K,

— the current intensity varies with time as $I = 10^6\tanh(2000t)$.

Characteristics of different computation subdomains are summarized in Table 1.

Using these input data and assuming that the initial temperature is T_0 and the magnetic field is zero, we performed calculations which gave qualitative and quantitative patterns of phenomena related to the diffusion of the electromagnetic field into the rails. The numerical-calculation parameters are as follows: the time step is 1 microsecond, the size of the computation grid is 65 × 65, and the relaxation parameter is 1.2.

Figures 1–3 show isolines of the module of the magnetic-induction vector $|B|$, current density j_x, and temperature increment $\Delta T = T - T_0$ for a symmetric profile. Figures 4–6 present these quantities in the corresponding order for an asymmetric profile.

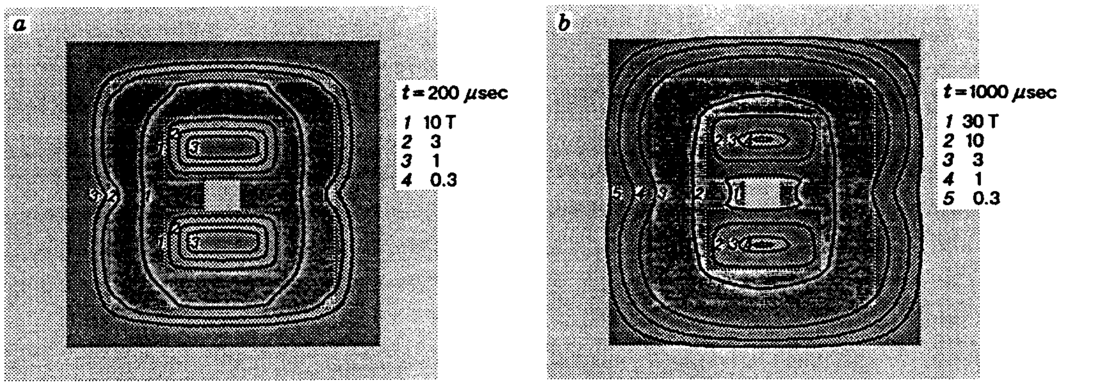

Fig. 1

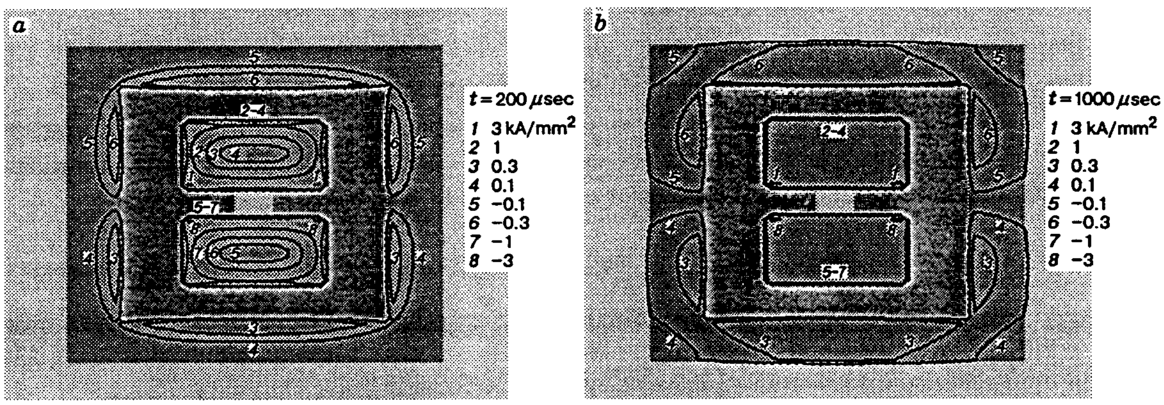

Fig. 2

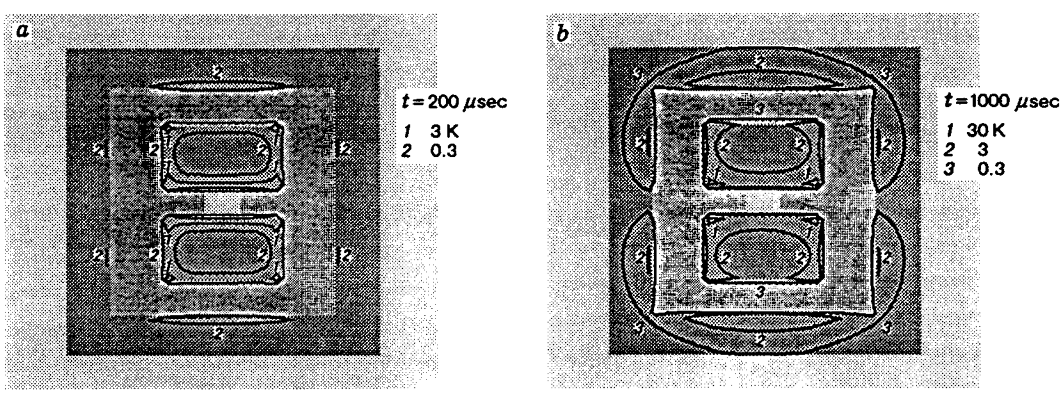

Fig. 3

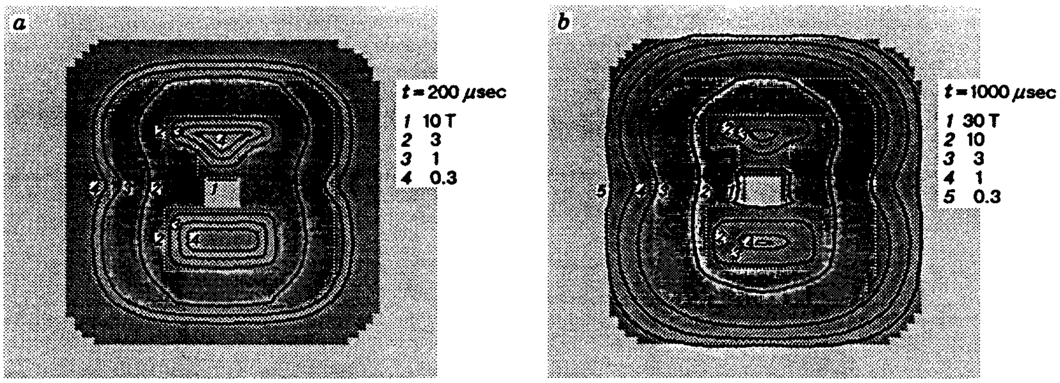

Fig. 4

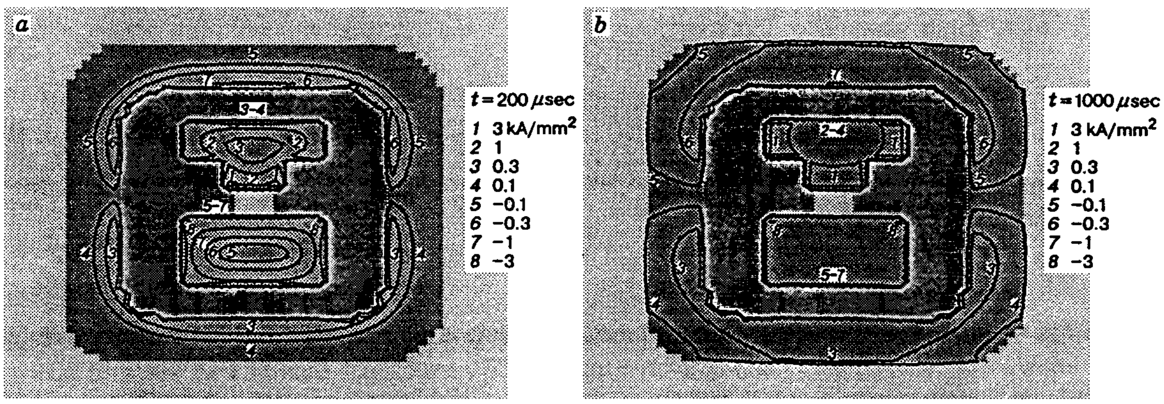

Fig. 5

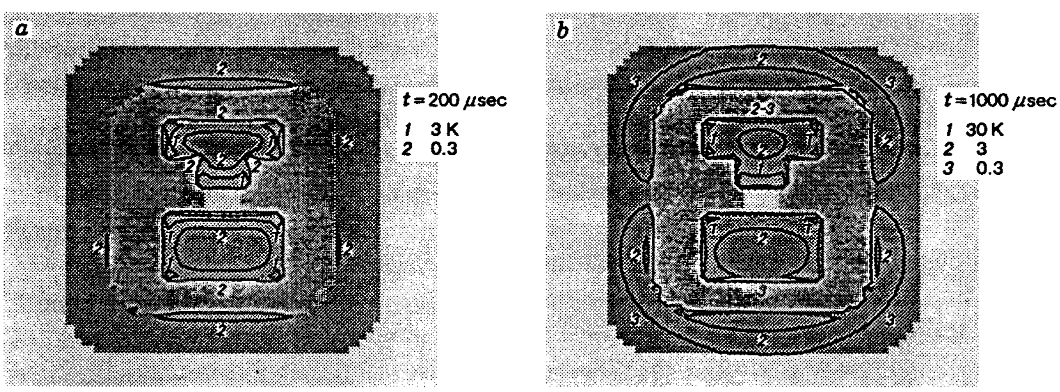

Fig. 6

TABLE 1

σ, S	ρ, kg/m^3	c_p, J/(kg·K)	λ, W/(m·K)
		Rails	
$2.5 \cdot 10^7$	8900	390	58
		Armature	
$2.5 \cdot 10^7$	8900	390	58
		Dielectric	
0	1000	1500	50

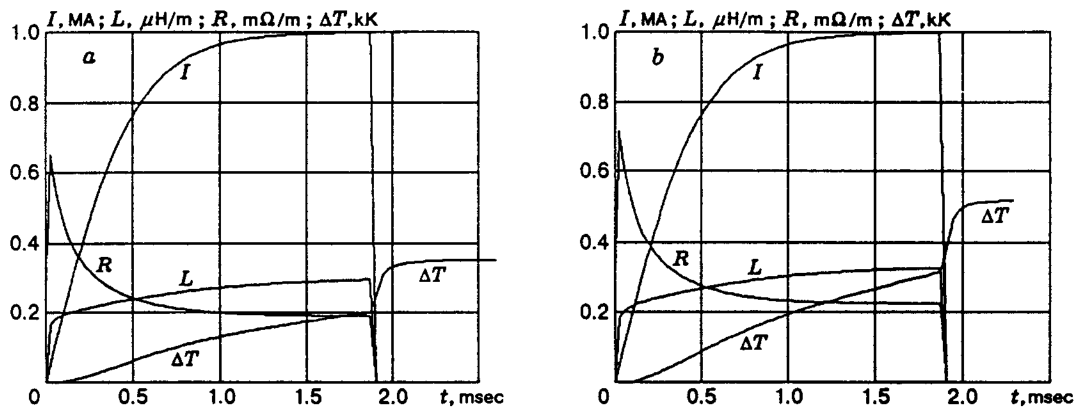

Fig. 7

Figure 7 shows the resistance R and inductance L per unit length, current intensity I and maximum temperature increment ΔT as functions of time for $t < 2200$ μsec (Fig. 7a refers to the symmetric profile, and 7b indicates the asymmetric profile). The calculation is carried out up to $t = 1900$ μsec with an increasing current intensity. Then, the circuit is assumed broken, the total current in the circuit is set equal to zero, and the decay of the magnetic field in the conductors is investigated. The resistance and inductance per unit length are not determined in the case of zero total current.

The qualitative pattern of diffusion of the magnetic field into the rails in the profile plane is as follows. In an initial stage, the magnetic field builds up linearly and is located mainly in the space between the rails and the armature. An electric current flows in the vicinity of the surface of the rails and in those places of the armature which are closest to the rails [3–6, 13]. The maximum magnitude of magnetic induction is observed in the railgun channel. It decreases rapidly at the entrance of the space between the rails and deep inside the conductors. The current density is maximal at the corners of the conductors, and primarily at the corners of the rails. A comparison of the results for the symmetric and asymmetric profiles shows that this takes place only for the exterior corners. The temperature increases primarily on the rail surface facing the railgun channel, and it is notably high at the corners. The pattern changes somewhat as the current increases and reaches a steady state. The magnetic field penetrates into the rails and the armature. The maximum magnetic induction remains in the space between the rails in the railgun channel, as before. The electric current also penetrates into the conductors and tends to fill them. The temperature rise becomes noticeable not only at the corners of the conductors, but also over their entire surface (with no penetration deep into the conductors, as before). The resistance per unit length of the system decreases, and the inductance increases with diffusion of the magnetic field. Both quantities tend to a value that corresponds to the steady case.

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